ON DEPLOYING REPEATERS IN CDMA SYSTEMS FOR TRAFFIC HOT-SPOTS: AN ANALYTICAL CHARACTERIZATION

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ABSTRACT

This paper describes and models a multi-cellular multi-service CDMA system with repeaters in an environment with traffic and spatial non-uniformities. An analytical model is presented and expressions for the required transmitted power and outage probability are derived for the reverse link. Results are focused on determining the maximum load factor that could be acceptable in order to limit the outage probability.

I. INTRODUCTION

The main goal of a CDMA network deployment is the provision of reliable coverage to mobile users whereas achieving high capacity. Both objectives are tightly coupled in CDMA based systems and so special care must be taken during the network deployment phase. Additionally, third generation (3G) scenarios are envisaged to be deeply non-homogeneous in terms of spatial distribution of users as well as in terms of service distribution. The combination of such non-homogeneities and the inherent dependence of capacity on the interference may lead to a reduction of the total capacity. Deployment solutions to cope with traffic and service non-homogeneities (such as microcells and repeaters) have been proposed in the literature [1]-[4]. Microcells allow an improvement of the capacity due to the reduction of the average loss between users and serving base stations. Moreover, their deployment does not imply a significant increase of the interference since low power levels are usually transmitted in these microcells. Despite advantages provided by microcells to face non-homogeneous scenarios, repeaters also come up as a cost efficient solution. Although repeaters were first addressed to cover dead zones [5][6], they have also been proved to be appropriate for high density areas coverage [3][4]. As with microcells, the inclusion of repeaters varies parameters such as statistics of distance between users and serving nodes or distance to interference sources. Repeaters, though, may cause undesired effects such as noise rise, specially in the uplink (the aggregate of the thermal noise of all repeaters) [3][4]. Yet, it has been shown in the literature that an appropriate configuration increases the capacity of the system [3].

QoS (Quality of Service) offered to users connected to the network does not only depend on the deployment configuration, but on the RRM strategies. Once the number of nodes and their locations have been determined, the dynamics of the network are monitored and controlled to maintain an appropriate QoS level. The CAC (Call Admission Control) is the first mechanism to avoid an excessive increase of the load. A maximum load threshold ($\eta_{\text{max}}$) is set, and all those admission requests whose acceptance implies a load increase above $\eta_{\text{max}}$ are rejected. In this paper an analytical model is presented to analyse the effect of repeaters in a multi-cell multi-service environment with traffic and spatial non-uniformities. Furthermore, it will allow working out the maximum acceptable load ($\eta_{\text{max}}$) to prevent overload that causes outage degradation. Then, the maximum load values are employed as inputs to tune the CAC.

The remainder of the paper is organized as follows: The problem formulation is presented in section II. In section III an analytical model is proposed and an expression for outage probability is derived. Finally section IV presents some results and section V summarizes the conclusions.

II. PROBLEM FORMULATION

The traffic of the proposed scenario is distributed among K round shaped layers ($L_i$, $0 \leq i \leq K-1$), each of them characterized by its distance to the coordinates’ origin ($\delta_i$), the angle ($\theta_i$), its radius ($\rho$) and the proportion of the overall number of users contained within it ($\alpha_i$). These traffic layers are spread all over the area under study. Likewise, the whole scenario is divided into M cells ($C_m$, $0 \leq m \leq M-1$). Each of these cells can be composed of more than a single node. Thus, cell $C_m$ has got $N_m$ nodes denoted as $\Omega_{m,j}$ ($0 \leq j \leq N_m-1$). These nodes (a base station and $N_m-1$ repeaters) are also characterized by their location with respect to the coordinates’ origin: distance to the origin ($\Delta_m$) and angle with the axis ($\Theta_m$), being $m$ the cell to which they belong and $j$ the node identifier (Fig. 1). All repeaters of a particular cell ($\Omega_{m,j}$ with $1 \leq j \leq N_m-1$) are connected to the donor base station ($\Omega_{m,0}$) through a radio link. Each base station-repeater link is characterized by propagation losses (including all effects derived from the radio interface) and the gain of the antennas. The final coupling gain between the donor base station and the $j$-th repeater is denoted as $\phi_{m,j}$. Taking into account that $\Omega_{m,0}$ is the base station of cell $C_m$, $\phi_{m,0}=0$dB. Thermal noise is another crucial aspect that plays a key role in the performance of the system when one or several repeaters are deployed within the cell. Hereafter noise power of repeaters will be referred with respect to the base station noise power. Thus, the thermal noise power of node $j$ is expressed as $P_{N_{m,j}}=P_{N_{m,j}}\cdot P_{N_{m,0}}$, being $P_{N_{m,0}}$ the thermal noise of the node $\Omega_{m,0}$, the base station ($\beta_{m,0}=1.0$). Usually, repeaters tend to be noisier than base stations and so $\beta_{m,j} \geq 1$.
III. ANALYTICAL MODEL

A. Total Loss Distribution

The number and location of nodes, as well as parameters such as their coupling gain ($\phi_{m,i}$), have an important effect on the total loss ($Z$) observed by users within each cell. Focusing on the node $\Omega_{m,i}$ of a generic cell $C_m$, a user will be connected to $\Omega_{m,i}$ only if $Z_{m,i} < Z_{n,j}$ for all the nodes in the scenario, including those nodes belonging to the same cell ($C_m$) and to the rest of cells ($C_n \setminus \{0,..,M-1\}$), $n \neq m$.

The total loss observed by users with respect to cell $m$ through node $i$ is obtained by deriving the distribution function, the probability density function of the total loss in cell $C_m$ is the following:

$$Z_{m,i}(dB) = Y_0 + \gamma \log \left( \frac{r_{m,i}}{r_{b}} \right) + \chi_{m,i}(dB) - \phi_{m,i}(dB)$$

where $Z_{m,i}$ is the total loss of a user connected to node $\Omega_{m,i}$, $r_{m,i}$ is the distance from node $\Omega_{m,i}$ to the user location, $\chi_{m,i}$ stands for the lognormal shadowing and $\phi_{m,i}$ for the coupling gain between node $\Omega_{m,i}$ and the base station to which it is connected ($\Omega_{m,i}$). $Y_0$ and $\gamma$ are constants and depend on the environment.

Taking (1) into account, and assuming hard handover, a user will be connected to $\Omega_{m,i}$ if $Z_{m,i} < Z_{n,j}$

$$\{\forall n \in [0,..,M-1], \forall j \in [0,..,N-1], (n,j \neq m,i)\}$$

It may also be expressed as:

$$\chi_{m,i}(dB) < B_{m,i}^{n,j} + A_{m,i}^{n,j} + \chi_{n,j}$$

$$\{\forall n \in [0,..,M-1], \forall j \in [0,..,N-1], (n,j \neq m,i)\}$$

with $A_{m,i}^{n,j} = \phi_{m,i}(dB) - \phi_{n,j}(dB)$ and $B_{m,i}^{n,j} = \gamma \log \left( \frac{r_{m,i}}{r_{n,j}} \right)$.

The CDF (Cumulative Distribution Function) of the total loss of cell $C_m$ is then the following:

$$F_{Z_m}(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{r,\varphi}(r,\varphi) p(Z_m \leq z) \, dr \, d\varphi$$

being $Z_m$ the total loss observed by users within cell $C_m$ regardless of the node through which they are connected to $C_m$ and $r$ and $\varphi$ are the distance and angle with respect to the coordinates’ origin. First, let define the probability event $G_{m,i}$ as:

$$G_{m,i} = \{\chi_{m,i} < B_{m,i}^{n,j} + A_{m,i}^{n,j} + \chi_{n,j}\}$$

Now, the probability $p(Z_m < z)$ is defined as:

$$p(Z_m \leq z) = \sum_{i=0}^{N-1} \left[ P_{m,i}^1 \cdot P_{m,i}^2 \right]$$

where $P_{m,i}^1$ and $P_{m,i}^2$ are as follows:

$$P_{m,i}^1 = \frac{p \left( Z_{m,i} \leq z, G_{m,i} \right)}{p \left( G_{m,i} \right)}$$

$$\{\forall n \in [0,..,M-1], \forall j \in [0,..,N-1], (n,j \neq m,i)\}$$

Then, $p(Z_m < z)$ remains as:

$$p(Z_m \leq z) = \sum_{i=0}^{N-1} \left[ \sum_{p=0}^{M-1} p \left( Z_{m,i} \leq z, G_{m,p} \right) \right]$$

$$\{\forall n \in [0,..,M-1], \forall j \in [0,..,N-1], (n,j \neq m,i)\}$$

Taking into account that:

$$p \left( Z_{m,i} \leq z, G_{m,i} \right) p \left( \chi_{m,i} < z - Y_0 - \gamma \log \left( \frac{r_{m,i}}{r_{b}} \right) - \phi_{m,i} \right) =$$

$$= \sum_{i=0}^{N-1} \left[ \prod_{p=0}^{M-1} \prod_{j=0}^{N-1} Q \left( \frac{\chi_{m,i} - B_{m,i}^{n,j} - A_{m,i}^{n,j}}{\sigma} \right) \phi_{m,i} \right]$$

and

$$p \left( \bigcup_{p=0}^{M-1} G_{m,p} \right) = \sum_{i=0}^{N-1} \left[ \prod_{p=0}^{M-1} \prod_{j=0}^{N-1} Q \left( \frac{\chi_{m,i} - B_{m,i}^{n,j} - A_{m,i}^{n,j}}{\sigma} \right) \phi_{m,i} \right]$$

where Q stands for the normalized form of the cumulative normal distribution function:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} \, dt$$

Finally, according to (3), (9) and (10) and knowing that pdf is obtained by deriving the distribution function, the probability density function of the total loss in cell $C_m$ is the following:

$$f_{Z_m}(z) = \sum_{i=0}^{N-1} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{r,\varphi}(r,\varphi) \, dr \, d\varphi \right]$$
where \( \phi_{m,i} = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x_{m,i}-\mu)^2}{2\sigma^2}} \prod_{j=1}^{n} Q\left(\frac{x_{m,i,j}-Y_{s,j}-\log(r_{i,j})}{\sigma}\right) \)

As expected, the total loss is dependant on the distribution of users. \( f_{R_m,n}(r_m,q_m) \) is the probability density function of users, which is dependant on the distance \((r_m)\) and the angle \((q_m)\), with respect to the node \( \Omega_{m,i} \) location. In fact, \( f_{R_m,n}(r_m,q_m) \) is the particular expression of \( f_{R,q}(r,q) \) when node \( \Omega_{m,i} \) is taken as the coordinates origin.

Regarding traffic distribution, let consider a circular traffic layer with its specific parameters as the described before. The probability density function of the distance for layer \( L_i \) is the following:

- If \( \delta_i < \rho_i \):

  \[
  f_i(r) = \frac{2r}{\rho_i^2} \left[ \pi - 2 \arcsin \left( \frac{\delta_i^2 + r^2 - \rho_i^2}{2r\delta_i} \right) \right]
  \]  
  \[
  \text{In the range } 0 \leq r \leq \rho_i - \delta_i .
  \]  

- If \( \delta_i \geq \rho_i \):

  \[
  f_i(r) = \frac{r}{\rho_i^2} \left[ \pi - 2 \arcsin \left( \frac{\delta_i^2 + r^2 - \rho_i^2}{2r\delta_i} \right) \right]
  \]  
  \[
  \text{In the range } \rho_i - \delta_i \leq r \leq \rho_i + \delta_i .
  \]  

The probability density function of the angle is as follows:

- If \( \delta_i < \rho_i \):

  \[
  f_i(\theta) = \frac{1}{\pi\rho_i} \left[ \delta_i \cos(\theta) - \frac{\delta_i^2}{2} + \delta_i \cos(\theta - \delta_i) - \delta_i^2 \sin^2(\theta - \delta_i) \right]
  \]  
  \[
  \text{In the range } 0 \leq \theta < 2\pi .
  \]  

- If \( \delta_i \geq \rho_i \):

  \[
  f_i(\theta) = \frac{2}{\pi\rho_i} \left[ \delta_i \cos(\theta - \delta_i) - \delta_i^2 \sin^2(\theta - \delta_i) \right]
  \]  
  \[
  \text{In the range } \delta_i - \arcsin \left( \frac{\rho_i}{\delta_i} \right) \leq \theta \leq \delta_i + \arcsin \left( \frac{\rho_i}{\delta_i} \right).
  \]

Finally, the joint probability density function \( f_{R,q}(r,q) \) of the i-th traffic layer is obtained by multiplying the two marginal functions \((r, \theta)\) are independent variables). Taking into account that traffic is distributed among K layers, each of them with a proportion of the total traffic equal to \( \alpha_n \), the total probability density function is:

\[
 f_{R,q}(r,q) = \sum_{n=1}^{K} \alpha_n \cdot f_{R,n,q}(r,q)
\]

Therefore, the pdf of the total loss could also be expressed as:

\[
 f_{L_m}(z) = \sum_{n=1}^{K} \alpha_n \cdot f_{m,n}(z) = \sum_{n=1}^{K} \kappa_{m,n} \cdot f_{L_m}(z)
\]

where

\[
 \kappa_{m,n} = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x_{m,n}-\mu)^2}{2\sigma^2}} \prod_{j=1}^{n} Q\left(\frac{x_{m,n,j}-Y_{s,j}-\log(r_{i,j})}{\sigma}\right)
\]

(19)

B. Transmitted Power distribution and Outage Probability

Once analysed the probability density function of the total loss in a multi-cell non-uniformly distributed traffic scenario, the impact of these scenarios can be found out in terms of required transmitted power, or in other words in terms of outage probability. It has been shown in the literature [3] that uplink becomes the limiting link when repeaters are deployed. The increase of the noise figure caused by repeaters (usually noisier than base stations) makes the uplink become a bottleneck. Then, the uplink required transmitted power for a user connected to cell \( C_m \) through the base station or any of the repeaters is:

\[
 P_r = Z_m \left[ 1 - \frac{\sum_{n=1}^{N_m} \phi_{m,n}}{\sum_{n=1}^{N_m} \beta_{m,n} \phi_{m,n}} \right] \frac{W}{W + 1} + 1
\]

(23)

where \( W \) the total bandwidth, \( R_b \) the transmission bit rate, \( (E_b/N_0) \) the energy per bit versus the noise spectral density and \( \eta \) the uplink load factor defined as:

\[
 \eta = 1 - \frac{\sum_{n=1}^{N_m} \beta_{m,n} \phi_{m,n}}{\sum_{n=1}^{N_m} \beta_{m,n} \phi_{m,n} + 1} + \text{INTER} + \text{INTRA}
\]

(24)

It is worth noting that the required transmitted power pdf will depend not only on total loss but on statistics of service usage and parameters that characterize each of the considered services. Let identify a set of \( Q_k \) services \((S_k \text{ with } 0 \leq s \leq 1)\). \( \rho_{i,q} \) denotes the proportion of users with the q-th service in layer \( L_i \). In general, the traffic source is not continuously transmitting packets within a session and some activity periods alternate with inactivity periods (e.g. reading time during a WWW download or silence periods in speech calls). Then, the activity factor for the q-th service \( (s_q) \) is defined as the proportion of time in which a user is transmitting for this service taking into account session and intersession time. With regard to activity periods, in which a certain user is transmitting data through the air interface, there may exist several possibilities for each service to transmit the data flow, characterized essentially by a transmission bit rate, a channel.
code and a required (Eb/No). Following the 3GPP terminology, these possibilities will be denoted as TF (Transport Format) hereafter. Then, the set of TFs is denoted as TFS (Transport Format Set). The set of possible bit rates for the q-th service are then: Rb_{q,i} (0≤i≤TFS_q-1) and the corresponding (Eb/No): (Eb/No)_{q,i} with 0≤i≤TFS_q-1. p_{q,i} is defined as the probability that the q-th service uses the i-th TF, which depends on the specific MAC algorithm that executes the Transport Format selection in the uplink. Making use of expressions developed in a previous work [7], it is possible to express the probability density function of the required transmitted power as a function of total loss pdf and services usage probabilities. Thus, the pdf of the required transmitted power (f_{P_{\text{req}}}(p)) of users connected to cell C_m is as follows:

\begin{equation}
    f_{P_{\text{req}}}(p) = \frac{1}{\sum_{k=0}^{TFS_q-1} \sum_{i=0}^{K_1-1} \sum_{j=0}^{\epsilon} \lambda_k p_{i,j} f_{P_{\text{req}}}(p - \zeta_{q,j})}
\end{equation}

with

\begin{equation}
    \zeta_{q,j} = 10 \log \left( \frac{P_{\text{req}}}{\sum_{k=0}^{TFS_q-1} \sum_{i=0}^{K_1-1} \sum_{j=0}^{\epsilon} \lambda_k p_{i,j} f_{P_{\text{req}}}(p - \zeta_{q,j})} \right)
\end{equation}

for j = 0,...,TFS_q-1.

Finally, and for a given load factor, the total outage probability (\mu) expression in cell C_m results in:

\begin{equation}
    \mu = \frac{\sum_{k=0}^{TFS_q-1} \sum_{i=0}^{K_1-1} \sum_{j=0}^{\epsilon} \lambda_k p_{i,j} \int_{0}^{\infty} f_{P_{\text{req}}}(p - \zeta_{q,j}) dp}{\sum_{k=0}^{TFS_q-1} \sum_{i=0}^{K_1-1} \sum_{j=0}^{\epsilon} \lambda_k p_{i,j}}
\end{equation}

IV. RESULTS

The considered scenario is composed of a central cell (where all results are collected) and 6 neighbouring cells that belong to the first interference ring (Fig. 2). Distance between neighbouring cells is 2000m. Parameters of all base stations and of the propagation model are listed in Table 1 and Table 2. With regard to traffic distribution, two layers are considered, the first one with a radius \(r_0=3000\)m and centred in the central base station (\(\Delta=0\)). The second traffic layer has a radius \(r_1=100\)m and its distance to the base station is \(\Delta=850\). There are also two services with parameters and services’ distribution as shown in Table 3 and Table 4. 50% of users will employ service 1 and 50% of the users service 2 in both traffic layers. In Service 1, the first three transport formats are used with a probability of 0.2 (p_{1,0}=0.2, p_{1,1}=0.2, p_{1,2}=0.2) whereas the last transport format has a probability p_{1,3}=0.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_{\text{req}}</td>
<td>-103 dBm</td>
</tr>
<tr>
<td>(Y_0)</td>
<td>128.1</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>37.6</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>3 dB</td>
</tr>
</tbody>
</table>

Table 1. Propagation parameters.
will also be attenuated by this gain. The combination of these two facts implies that, for very low gains, results are the same than results of the case without repeaters.

![Graph](image)

Fig. 3. Maximum load associated to a 2% outage probability as a function of the coupling gain.

Results plotted in Fig. 3 have been obtained with $\alpha_0=0.9$ and $\alpha_1=0.1$. Yet, it has been checked that $\eta_{\text{max}}$ is higher when the density of traffic layer $L_1$ with respect to layer $L_0$ rises ($\alpha_1$ grows and $\alpha_0$ falls).

Fig. 4 depicts results obtained when repeater and traffic layer $L_1$ are placed in A and repeater’s noise is varied. The base station has thermal noise $P_N=-103\text{dBm}$ (Table 1) and the repeater has $P_N=-103\text{dBm}$ ($\beta=1$) or $P_N=-100\text{dBm}$ ($\beta=2$) respectively.

The effect of the repeater noise is more noticeable when noise level is increased (Fig. 4), and the maximum allowable load factor decreases as the noise grows. Yet, for low coupling gain values, differences in noise level are compensated.

![Graph](image)

Fig. 4. Maximum load for different repeater noise levels ($\mu_{\text{max}}=2\%$) as a function of the coupling gain.

V. CONCLUSIONS

An analytical model has been proposed to evaluate the maximum allowable uplink load factor in a multi-cell (including repeaters) multi-service WCDMA scenario with non-uniform traffic spatial distributions. The outage probability as a function of the load factor has been obtained in order to assess the appropriate maximum admission load in the described scenarios. It has been shown that the proper selection of the load threshold ($\eta_{\text{max}}$) is dependant on traffic and service distribution parameters as well as on the deployment of network. The presented model allows determining these load values for any scenario once a target outage probability ($\mu_{\text{max}}$) has been set.

VI. ACKNOWLEDGEMENTS

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